Shape Registration in Implicit Spaces Using Information Theory and Free Form Deformations

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Outline

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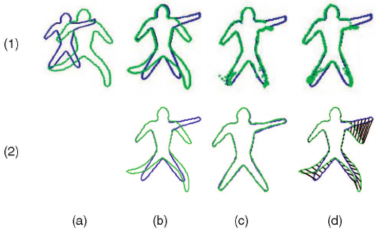
Shape Registration

Overview

- General alignment of two structures which may or may not be in the same orientation, view, or with different deformations.



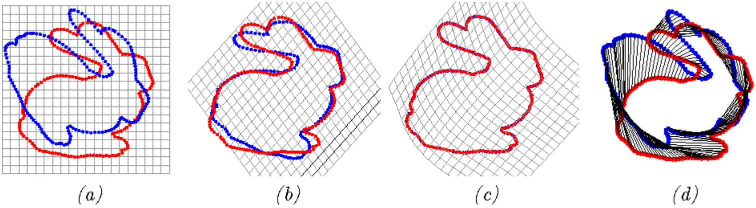


FIGURE – Shape registration

Shape Registration

Overview

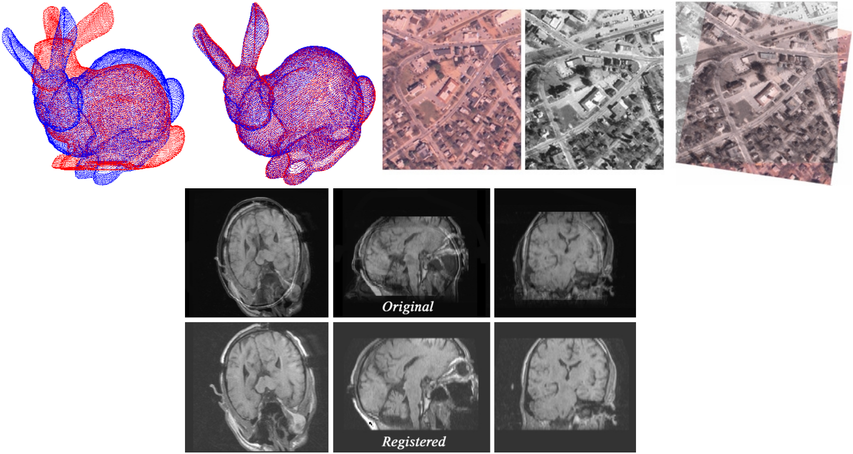


FIGURE – Shape registration

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Proposed Method

Overview of the Proposed Method Pipeline

Implicit Shape Representation

Represent the given two shapes in a convenient format in order to efficiently perform registration between them.

Global Registration

Using the implicit representation, perform registration on the global scale (with respect to the shapes).

Local Registration

Once global registration is finished, perform fine local registration to account for small, specific local changes that are missed during global registration.

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Implicit Shape Representation

Representing Shapes Implicitly without Parameterization

- Shapes are represented “implicitly", ie. without using any parameters.
- A Lipschitz Function $\phi : \Omega -> R^+$ denotes a distance transform of a shape S, such that :

$$\phi_S(x,y) \begin{cases} 0 & (x,y) \in S \\ +D((x,y),S) > 0 & (x,y) \in [R_s] \\ -D((x,y),S) < 0 & (x,y) \in [\Omega - R_s] \end{cases} \tag{1}$$

- Where $D((x,y),S)$ is the minimum Euclidean Distance between (x,y) and the shape S.

Implicit Shape Representation

Graphical Intuition

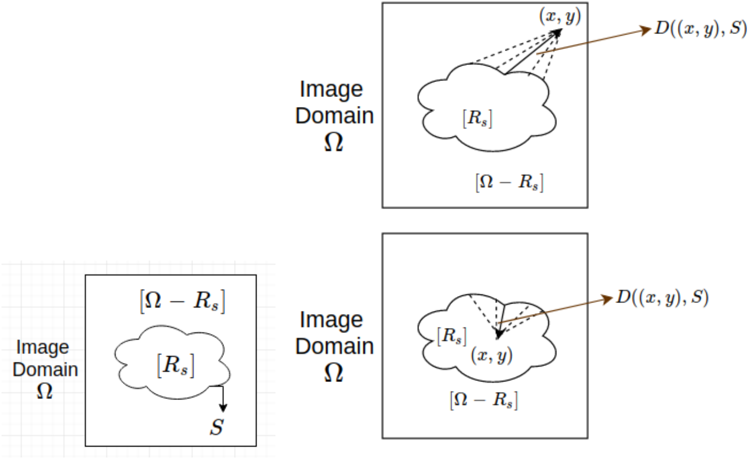

FIGURE – Regions in image domain Ω

Implicit Shape Representation

Graphical Intuition

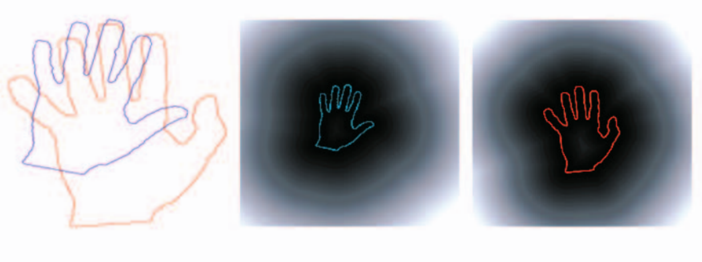

FIGURE – Implicit representation of shapes in the form of a 2D image of distances

Outline

Mutual Information based Global Registration

Information Theoretic Objective Function for Global Registration : Notations

- The Implicit Shape Representation (ISR) is inherently translation/rotation/scale invariant.
- The ISRs of source ($\phi(D) \quad$ as $\quad f$) and target ($\phi(S) \quad$ as $\quad g$) are registered in using global criteria.
- The "fitness" criteria, or the objective to maximize in order to get a good registration is chosen as the Mutual Information between the two ISRs.
- Let Ω be a sampling domain in the image [1], and $A(\theta)$ be a transformation.
- Then, we have :

$$f_\Omega = \underbrace{f(\Omega)}_{\substack{\text{Gives back} \\ \text{all pixels in} \\ \text{sample domain } \Omega}} \quad \text{and} \quad g_\Omega^A = \underbrace{g(A(\theta;\Omega))}_{\substack{\text{Gives back} \\ \text{all pixels in} \\ \text{sample domain } \Omega \\ \text{transformed by } A(\theta;\Omega)}} \tag{2}$$

1. theoretically it can be full image, but practically it's a ring around the shape

Mutual Information based Global Registration

Mutual Information

$$MI(f_\Omega, g_\Omega^A) = \underbrace{\mathcal{H}[p^{f_\Omega}(l_1)]}_{\substack{\text{Entropy of the} \\ \text{distribution representing } f_\Omega}} + \underbrace{\mathcal{H}[p^{g_\Omega^A}(l_2)]}_{\substack{\text{Entropy of the} \\ \text{distribution representing } g_\Omega^A \\ \text{which is the} \\ \text{transformed source ISR using } A(\theta)}} - \underbrace{\mathcal{H}[p^{f_\Omega, g_\Omega^A}(l_1, l_2)]}_{\substack{\text{Entropy of the} \\ \text{joint distribution} \\ \text{representing } f_\Omega,\, g_\Omega^A}} \quad (3)$$

Mutual Information based Global Registration

Graphical Intuition

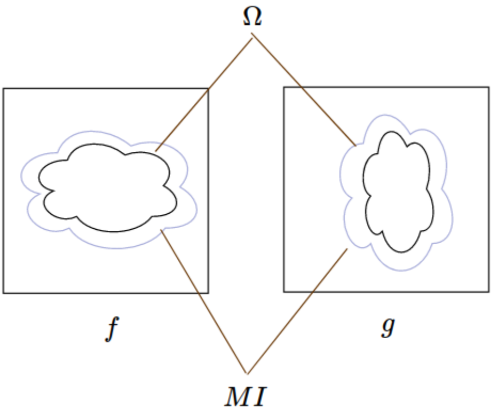

FIGURE – Sampling domain Ω around the actual shape, which is used in practice to improve running time.

Mutual Information based Global Registration

How to Model Distributions of the Pixels ?

- **Question :** Given a set of pixels (or points), how do we model it's probability density function (distribution) ?
- **Answer :** Represent pixels as a non-parametric Gaussian Kernel-Based Density Estimation model using the pixel values $\mathbf{x} = (x, y)$. Plug in the values into the following equations :

$$\begin{aligned} p^{f_\Omega}(I_1) &= \frac{1}{V(\Omega)} \int\int_\Omega G(I_1 - f(\mathbf{x}))d\mathbf{x} \\ p^{g_\Omega^A}(I_2) &= \frac{1}{V(\Omega)} \int\int_\Omega G(I_2 - g(A(\theta;\mathbf{x}))d\mathbf{x} \\ p^{f_\Omega, g_\Omega^A}(I_1, I_2) &= \frac{1}{V(\Omega)} \int\int_\Omega G(I_1 - f(\mathbf{x}), I_2 - g(A(\theta;\mathbf{x}))d\mathbf{x} \end{aligned} \tag{4}$$

Outline

Free Form Local Registration

Using FFD Technique to perform Local Registration

- Global registration is good enough for some applications, but not satisfactory for some others like medical imaging.
- There is a need to model local deformations of the shape, and for that, a modified Free Form Local Registration (FFD) is used, called the Incremental FFD (IFFD).
- The idea is to embed a mesh of “control points" on top of the shape, represented as :

$$P = \{P_{m,n}\} = \{(P^x_{m,n}, P^y_{m,n})\}; m = 1...M, n = 1...N \qquad (5)$$

Free Form Local Registration

Graphical Intuition

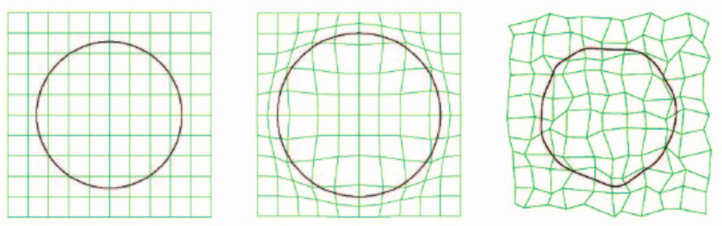

FIGURE – FFD control points embedding on a hypothetical “elastic membrane" which can be deformed arbitrarily

Free Form Local Registration

B-Spline Interpolation

- **Question :** What happens to the points (pixels) which are not control points ? In other words, how do you interpolate the control points to all pixels ?
- **Answer :** Use Cubic B-Splines Interpolation to determine displacement values of all pixels other than the control points.

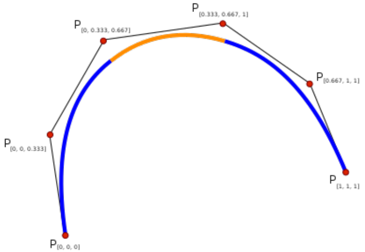

FIGURE – B-Spline interpolation in 1D

Free Form Local Registration

Optimization Criteria for Local Registration

- In order to guides the deformation of control points in order to get good local registration, an objective function is minimized.
- The Sum Squared Differences (SSD) acts as a “data-driven" objective function for optimization to get the “best" local registration between the source and target shapes.

$$E_{data}(\theta) = \int \int_{\Omega} (\phi_D(\mathbf{x}) - \phi_S(L(\phi; \mathbf{x})))^2 d\mathbf{x} \tag{6}$$

Multi-Scale Local Registration

Account for Finer Deformations in Registration

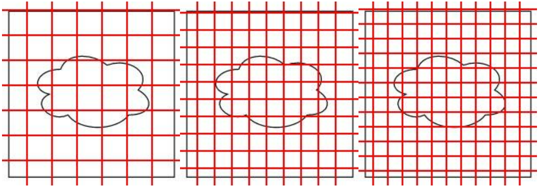

FIGURE – Multi-scale or multi-resolution registration by using finer control point grid. Each finer scale is initialized by the coarse scale grid, and then optimized again.

- The final deformation is then the sum of displacements of the pixels calculated through the scales, given as :

$$\delta L(\mathbf{x}) = \sum_{k=1}^{r} \delta L^{k}(\theta; \mathbf{x}) \tag{7}$$

Incorporate Feature Point Constraints

Adding Support for Feature Point Correspondences when Available

- Registration can be performed between any two arbitrary without knowledge of corresponding feature points between shapes.
- However, if there's prior knowledge about feature point correspondence between shapes, it can bolster registration.
- **Question :** How can we incorporate this prior knowledge to this registration framework ?

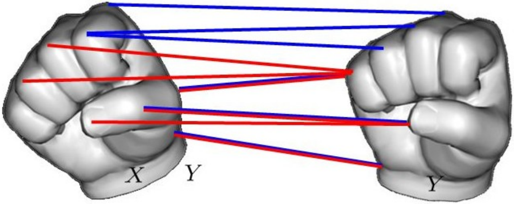

FIGURE – Point correspondence between shapes

Incorporate Feature Point Constraints

Plug-able Objective Term

- **Answer :** If there is correspondence information available, plug-in an objective term to the local registration energy term.
- The term is simply the Sum Squared Differences (SSD) of the feature points in transformed source shape, and the target shape.

$$E_{feature}(\theta) = \sum_{i} \left(L(\theta; x_{\hat{D}_i}) - \mathbf{x}_{S_i} \right)^2, \ i \in [1, n_c] \tag{8}$$

FIGURE – Results after incorporating feature point correspondences to aid registration of occluded shape with full shape

Outline

Statistical Modeling of Anatomical Structures

- **Objective :** Learn a compact representation that can capture variation in an anatomical structure of interest across the training data.
- **Requires :** Dense correspondence between feature points.

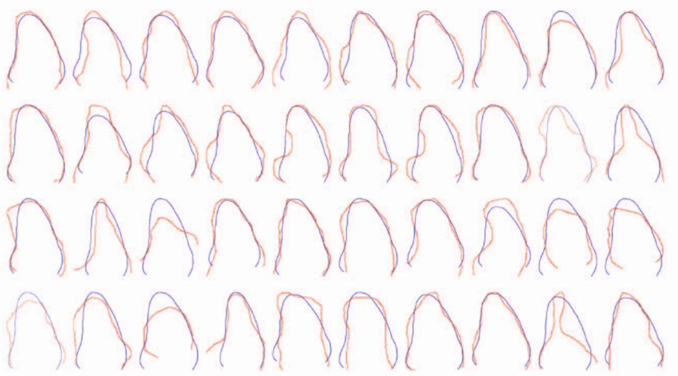

FIGURE – Registration between target mean shape of left ventricle (blue) and source shapes (red) for various views from ultrasonic images

Statistical Modeling of Anatomical Structures

FIGURE – Established correspondences, can be used to create Point Distribution Model (PDM) to capture statistics of elements across training examples.

- PDM Model captures statistical properties of the variation of training set, and can be used to generate more data, or fit a curve using the Active Shape Models method.

Outline

3D Face Registration

Extending Registration Framework to 3D on Natural Scenes

- **Objective :** Align, register and stitch 3D face scans captured from range scanners.

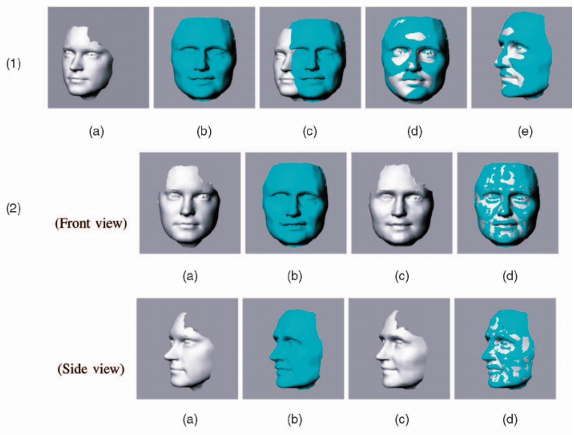

FIGURE – Face registration between two scans from range data, which are open and are in 3D. (1d, 1e) Translation/Scaling/Quaternion based rotation

Outline

Facial Expression Tracking in 3D Range Scan Data

Keypoint Matching in 3D

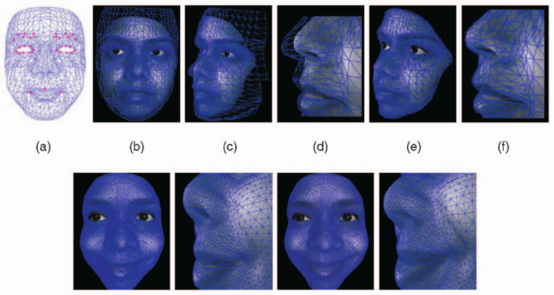

FIGURE – Registration of 3D mesh model of the face onto a given model.

Discussion and Critique

- The method is fast (given a small band around the shape that represents the domain Ω.
- Takes about 50ms for two 2D shapes to be registered.
- However, more comparison results could have been provided. (Currently only SC and ICP algorithms were compared against, on a single instance).

Thank You!

Questions ?