A Mathematical Tutorial on Reinforcement Learning

Presented By:
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Outline

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- Success is measured by a scalar reward signal.
- Goal: select actions to maximize future reward.

Reinforcement Learning

High level overview

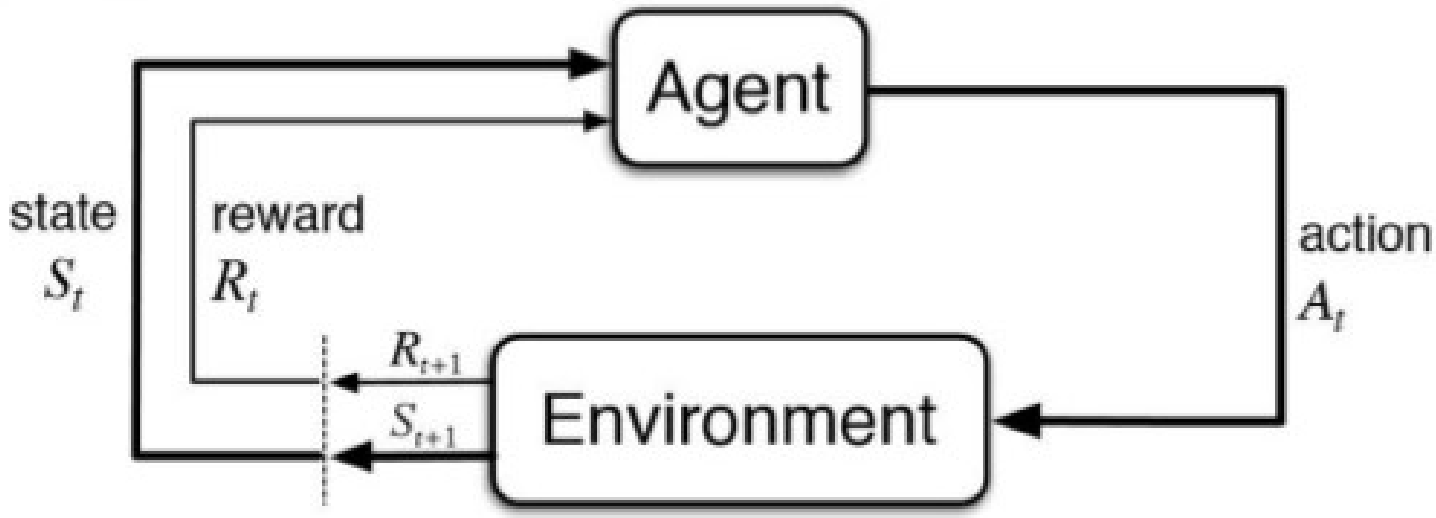

Figure: Agent-Environment interaction

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- Reward function provides the immediate, intrinsic desirability of environmental states.

Reward Function

Simple Visualization

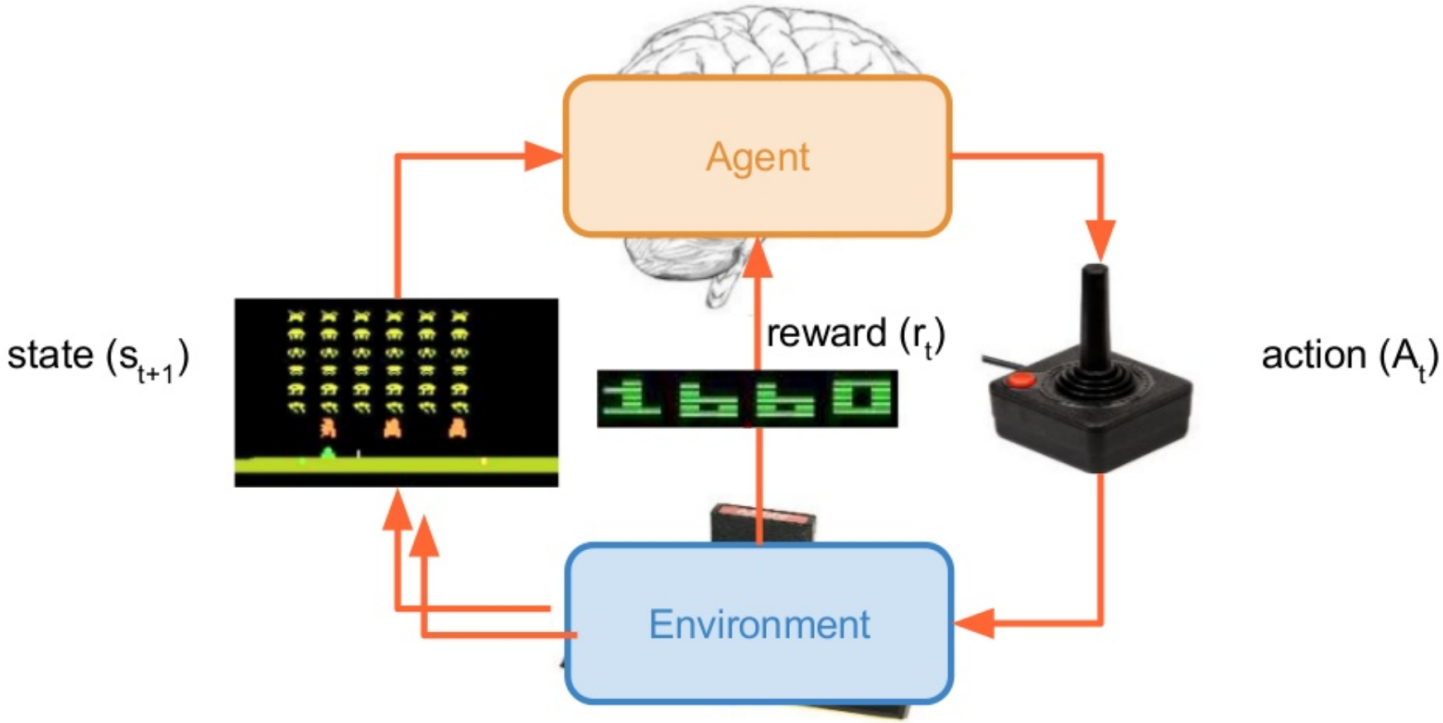

Figure: Atari Example for Reward Function

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- Value function estimates the worth of an environmental state in the long run.

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- Value function $\mathcal{V}(s_t, a_t)$ calculates this total reward quantity, given the current state and action agent takes.

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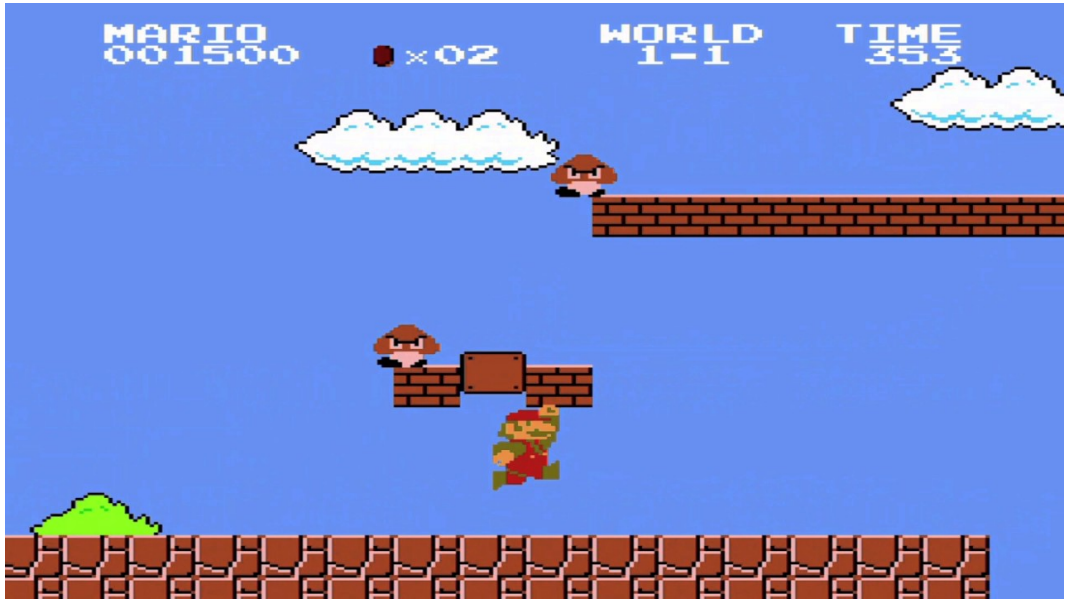

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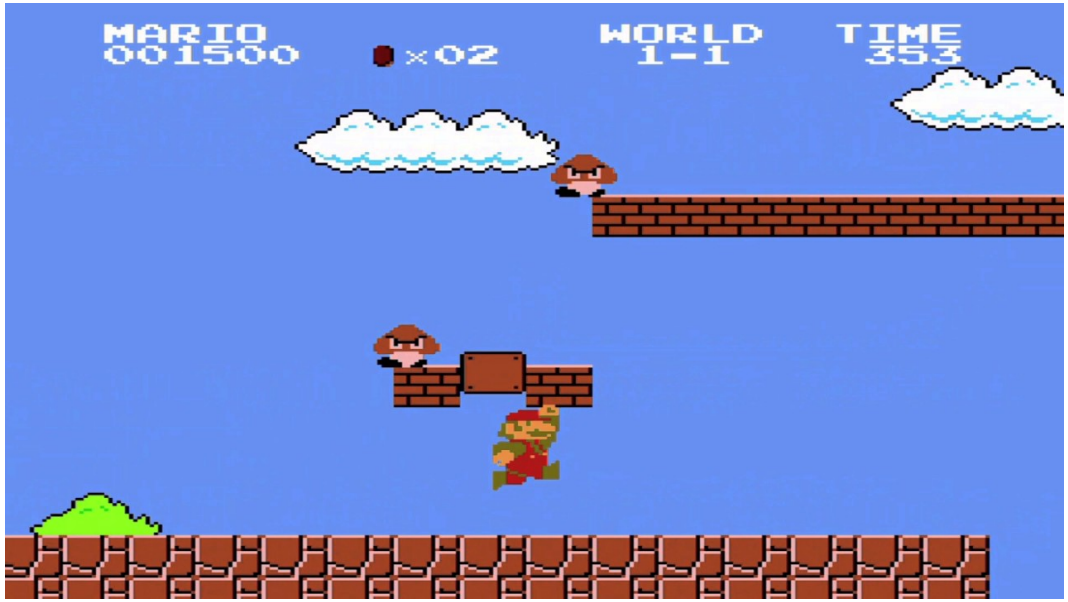

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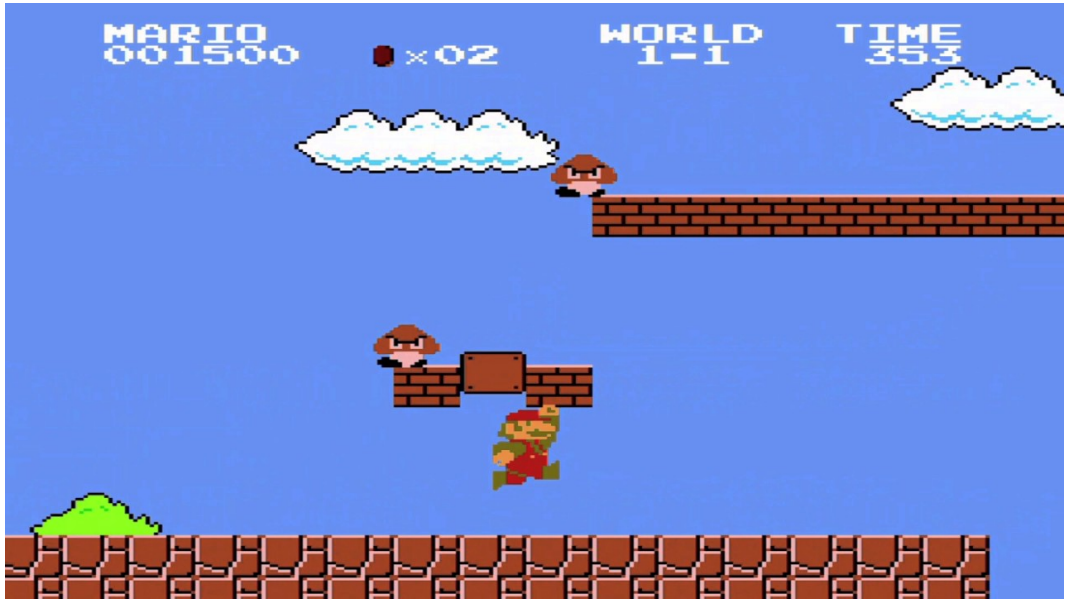

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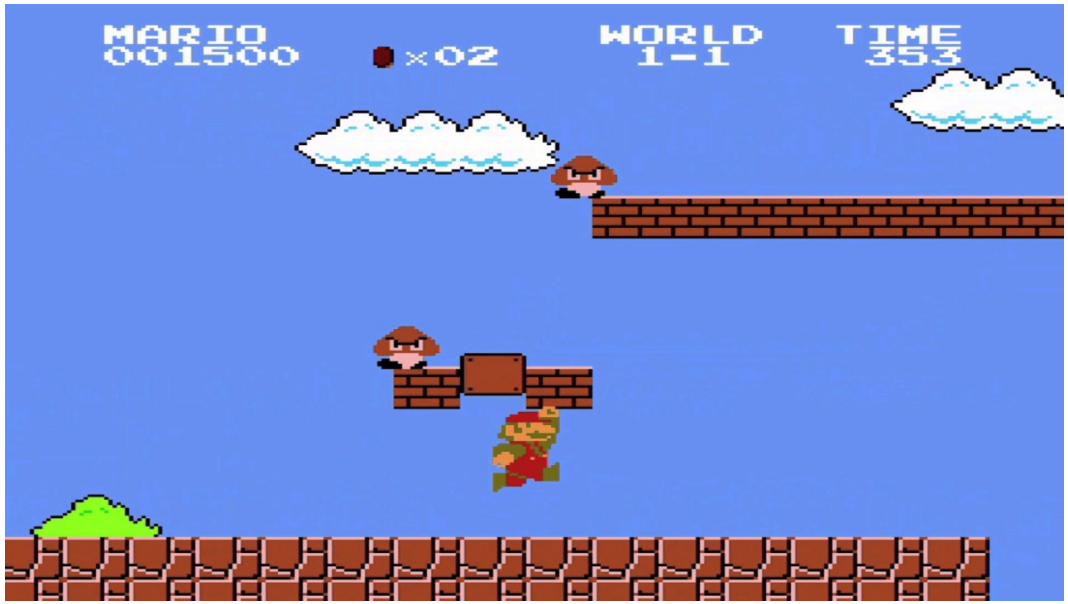

Figure: Super Mario Bros Atari emulator as an environment

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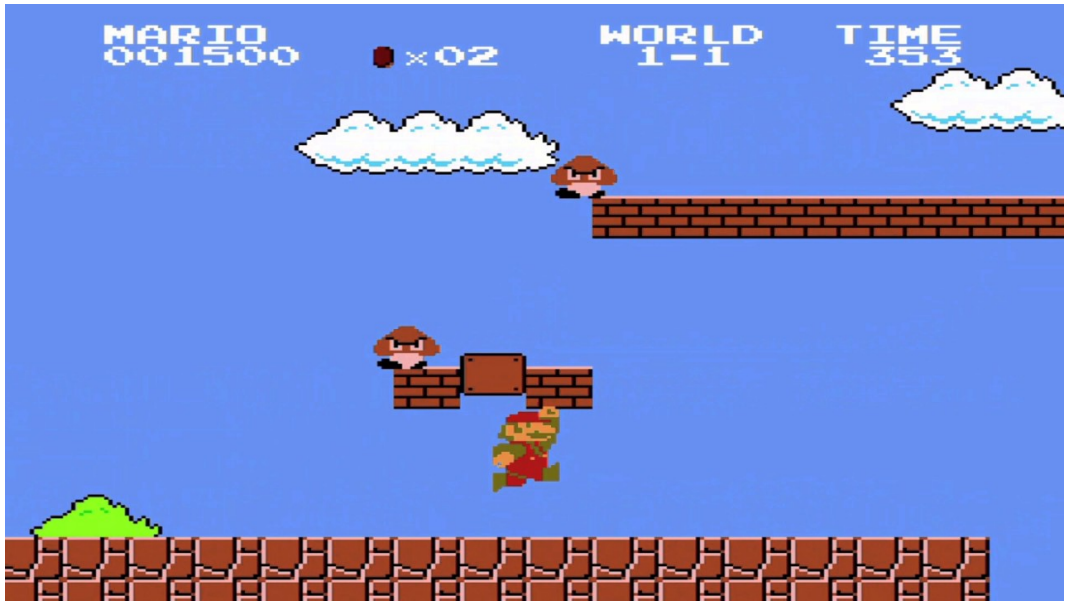

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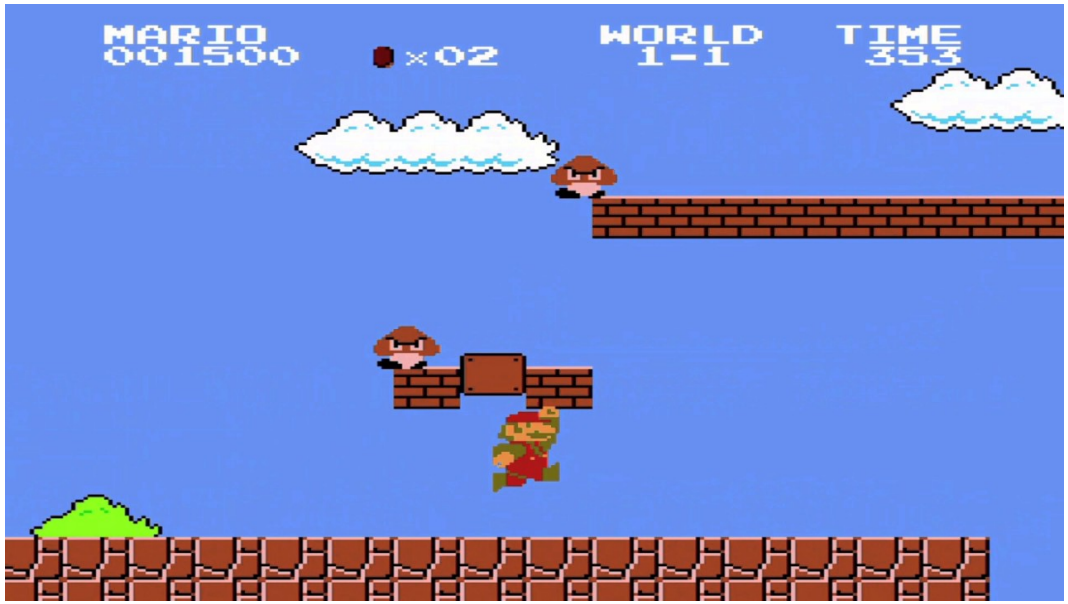

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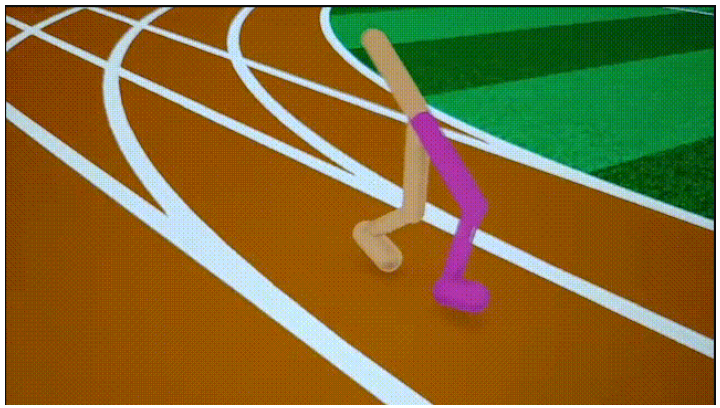

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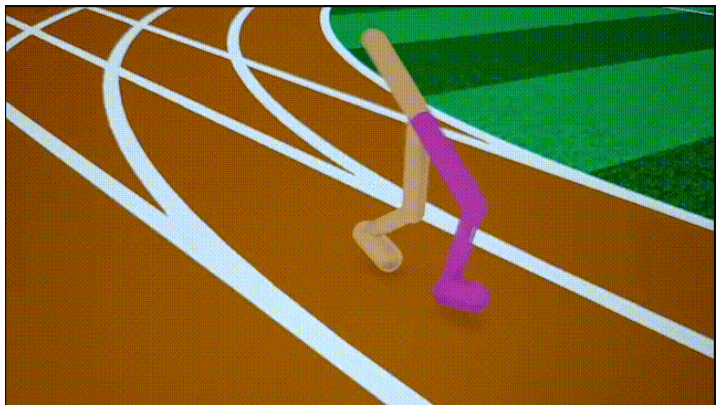

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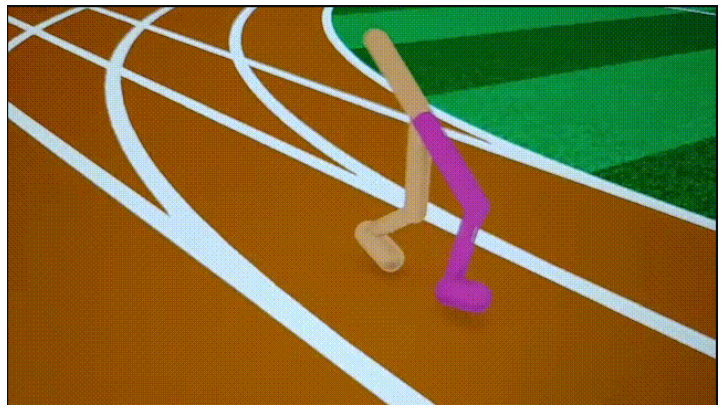

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- So what's a simple and intuitive way to do it?

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7. Return $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, ...s_{H-1}, a_{H-1}, r_{H-1})$

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    - In other words, it larger γ favours long term rewards, and the agent becomes willing to forgo short term rewards.

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$$J(\pi) = \mathbb{E}_\pi \left[\sum_{i=0}^{H-1} \gamma^i * r(s_t, a_t, s_{t+1}) | s_0 \right]$$

- In general: $\pi^* = argmax_\pi J(\pi)$

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- The concepts introduced here are general terminologies, and should be a good starting point to explore state-of-art literature in (Deep) RL.